Letters

A Method for Measuring the Refractive Index Profile of Thin-Film Waveguides

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Abstract—A method is described for determining the refractive index profile of an optical waveguide. An optimization algorithm has been employed to obtain the index profile from the measured reflection coefficient data. Several numerical experiments have been performed to prove the accuracy of the present method. Some of the results are included.

An important problem in integrated optical design is the determination of the refractive index profile of an optical waveguide, since the propagation and dispersion characteristics of the guide are strongly dependent on the profile function. A method for performing this measurement has recently been proposed by Tien *et al.* [1] and is based upon the use of a prism coupler.

This letter presents an alternative approach, based on computer processing of the angular scattering coefficient data obtained by reflecting a collimated light beam from a section of the waveguide. The method is applicable to both continuous and discretely stratified profiles. The geometry of the setup is shown in Fig. 1.

The computer processing of the measured reflection coefficient data proceeds as follows. As a first step, the reflection coefficient as a function of the incident angle θ_i of the collimated beam is computed for a profile function which is described in terms of a suitable set of unknown parameters. For the multilayer case, shown in Fig. 1(a), these parameters are taken to be the thickness $d_{1,2\ldots}$, loss factor $\sigma_{1,2,\ldots}$, and the dielectric constant $\epsilon_{1,2,\ldots}$ of the various layers.

For a normalized incident wave of unit magnitude, the complex reflection coefficient may be readily expressed in terms of these structural parameters and the incident angle θ_i as

$$\rho = (N_0 - Y_1)/(N_0 + Y_1), \qquad N_0 = \cos\theta_i/120\pi$$

$$Y_m = N_m \frac{Y_{m+1} + N_m \tanh \gamma_m d_m}{N_m + Y_{m+1} \tanh \gamma_m d_m}, \qquad m = 1,2,\cdots,M$$

$$Y_{M+1} = N_0, \qquad N_m = \gamma_m/(j\omega\mu_0), \qquad \gamma_m = \alpha_m + j\beta_m$$

$$\beta_m = (2\pi/\lambda)\{\epsilon_m - \sin^2\theta_i + [(\epsilon_m - \sin^2\theta_i)^2 + \sigma_m{}^2]^{1/2}\}^{1/2}/\sqrt{2},$$

$$\alpha_m = (2\pi/\lambda)^2\sigma_m/(2\beta_m).$$

An estimate of the unknown parameters d_m, σ_m, and $\epsilon_m, m = 1,2,\cdots$ is now obtained by making use of the parameter optimization technique on a digital computer. This requires the minimization of the so-called performance index function F which is defined by

$$F = \sum_{i=1}^{N} |\,\rho(\theta_i,\epsilon_m,\sigma_m,d_m) - \rho_0(\theta_i)\,|^2$$

$$\rho_0(\theta_i) = \text{measured reflection coefficient.}$$

It is evident that F represents the mean-square norm of the deviation between the measured angular variation of the reflection coefficient and the couputed values for the trial medium.

The above procedure for discrete media can also be applied to the case of continuous profiles with only slight modifications. Although the computation of the reflection coefficient for the trial medium is slightly more involved for a continuous medium, standard algorithms can still be used to evaluate this quantity for assumed profiles of $\epsilon(z)$ and $\sigma(z)$. An optimization algorithm is again used to minimize the performance index F defined in a like manner.

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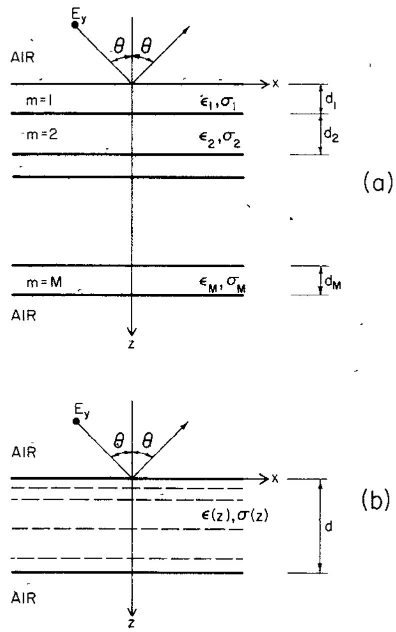

Fig. 1. Structure of optical waveguides and a method of scattering coefficient measurement.

TABLE I

1 Layer Case				(λ = 1.0 μm)		
	Actual Value			Computed Value		
	ε	σ	d(μm)	ε	σ	d(μm)
Case 1	2.5	0	5	2.50031	0	4.9996
Case 2	2.5	10^{-4}	5	2.5002	0.87×10^{-4}	4.9997

TABLE II

2 Layer Case				(λ = 0.6328 μm)					
	Actual Value			Computed Value					
				Trial 1			Trial 2		
Layer	ε	σ($\times 10^{-2}$)	d(μm)	ε	σ($\times 10^{-2}$)	d(μm)	ε	σ($\times 10^{-2}$)	d(μm)
1	2.5	0.1	1.2	2.4996	0.1197	1.2006	2.5004	0.1036	1.200
2	2.2	0.1	1000	2.1953	0.1126	1000.9	2.1998	0.1199	997.0

The accompanying tables show the results of computer simulation of the experimental scheme outlined above. The measured values of the complex reflection ρ_0 were simulated from computer evaluation of the reflection coefficient for a given set of ϵ_m, σ_m, and d_m. The optimization technique was then employed to recover the values of parameters. The results of the numerical experiments are quite favorable as may be seen by reference to Tables I and II.

The practical application of the method requires the measurement of the magnitude and phase of ρ_0. Although the magnitude of the reflection coefficient is obtainable in a fairly straightforward manner, extracting the phase information is usually not easy at optical frequencies. One of the proposed systems of obtaining the phase information is based on the use of holographic technique as

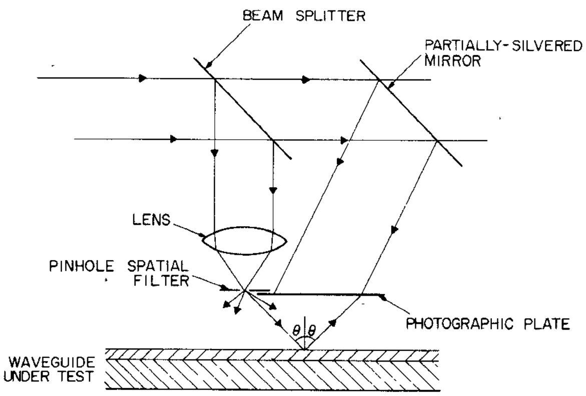

Fig. 2. Proposed setup for obtaining the phase information of the reflection coefficients.

shown, for instance, in Fig. 2. Using a beam splitter, collimated light from a laser is split into two beams, one of which is focused to generate a point source using a converging lens and a pinhole spatial filter. Rays diverging from this point source impinge on the waveguide surface at different incident angles as sketched in the diagram. After reflection, these rays are then made to interfere with a reference beam derived from the second beam via the use of a partially silvered mirror. The intensity of the reference beam may be controlled either by appropriately choosing the transmittance of the partially silvered mirror or via the insertion of an optical attenuator along the path of the beam.

The hologram formed in this manner contains the phase information on $\rho_0(\theta)$ for all $\theta_a < \theta < \theta_b$, where θ_a and θ_b are angles determined by the physical arrangement of the measurement system. This phase information can be extracted from the knowledge of the intensity distribution in the hologram via the use of a photodensitometer. This method has been successfully used by Stigliani *et al.* [2] for processing holographic recordings.

One restriction in this method is that the distance from the pinhole filter to the surface of the waveguide should be greater than $l \approx 2D^2/\lambda$ where D is the diameter of the pinhole. However, for $\lambda = 0.6\ \mu$m and $D = 10\ \mu$m, the minimum value of l is 0.3 mm which is easily attained. It should be emphasized once again that the method outlined above is only a proposed one and has yet to be verified experimentally.

Finally, it should be noted that the method presented in this letter is quite general and is equally well applicable to nonplanar geometries, e.g., optical fiber waveguides.

REFERENCES

[1] P. K. Tien, G. Smolinsky, and R. J. Martin, "Thin organosilicone films for integrated optics," *Appl. Opt.*, vol. 11, pp. 637–642, Mar. 1972.

[2] D. J. Stigliani, R. Mittra, and R. G. Semonin, "Spherical particle size analysis. Part I: Back scatter holography," *J. Opt. Soc. Amer.*, vol. 60, pp. 1059–1067, Aug. 1971.

Distributed Capacitance of a Thin-Film Electrooptic Light Modulator

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***Abstract*—An analytical method is described which determines the distributed capacitance of a thin-film electrooptic light modulator with parallel-strip electrodes. The capacitance is expressed in a variational form. The anisotropy of the crystal and the presence of surrounding air are considered. Numerical examples for the case of Li–Nb–O_3 are given.**

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I. INTRODUCTION

The development of active thin-film components is very important in constructing integrated optical devices. An electrooptic light modulator with parallel-strip electrodes made with the out-diffusion technique [1] appears to be a practical structure.

This letter describes an analytical method to determine the distributed capacitance, as a basic circuit parameter, of the above thin-film light modulator based on a variational method [2], [3].

II. A THIN-FILM ELECTROOPTIC LIGHT MODULATOR AND ITS DISTRIBUTED CAPACITANCE

The cross-sectional view of a thin-film light modulator reported by Kaminow *et al.* [1] is shown in Fig. 1. An important circuit parameter of this structure is the capacitance between the electrodes which limits the modulation bandwidth. Since the electrooptical crystal like Li–Nb–O_3 is anisotropic and the electric line of force between the parallel-strip are not straight, this anisotropy should be taken into account in the analysis of the capacitance. A similar structure with isotropic media as shown in Fig. 2 has been treated by a variational method in a previous paper [2] which we apply to the present case with some modification.

Suppose the electrodes are very long and thin compared with a and b. Then a basic equation to govern the potential distribution $\phi(x,y)$ is the two-dimensional Laplace's equation,

$$\epsilon_x \frac{\partial^2\phi}{\partial x^2} + \epsilon_y \frac{\partial^2\phi}{\partial y^2} = 0 \tag{1}$$

The charge density distribution on the electrodes can be approximated as

$$\rho(x,y) = f(x)\delta(y) \tag{2}$$

where $\delta(y)$ is Dirac's delta function.

By applying the Fourier transform

$$\tilde{\phi}(\beta,y) = \int_{-\infty}^{\infty} \phi(x,y) \exp(j\beta x)\, dx \tag{3}$$

to the potential in each medium and using continuity conditions, we can obtain the solution of $\tilde{\phi}(\beta,y)$. Instead of carrying out the inverse transform, the capacitance per unit length is expressed in a variational form in the transformed domain [3],

$$\frac{1}{C} = \frac{1}{\pi Q^2}\int_0^{\infty} [\tilde{f}(\beta)]^2 \tilde{g}(\beta)\, d\beta \tag{4}$$

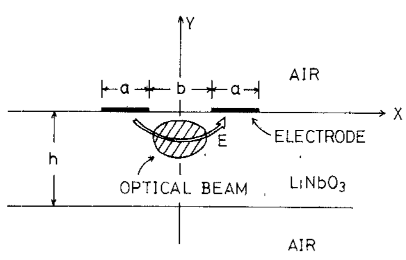

Fig. 1. An electrooptic light modulator structure proposed by Kaminow *et al.* [1].

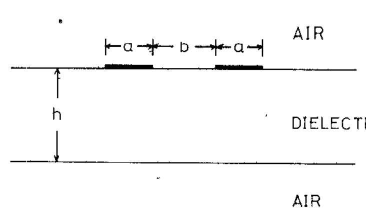

Fig. 2. Parallel-strip line printed on a dielectric sheet [2].